NEW YORK UNIVERSITY
INSTITUTE OF MATHEMATICAL SCIENCES
DIVISION OF ELECTROMAGNETIC RESEARCH

RESEARCH REPORT No. EM-61

ON DIFFRACTION BY AN INFINITE GRATING

by

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CONTRACT No. AF-19(122)-42

CEMBER 1953

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The research reported in this document has been made possible through support and sponsorship extended by the Air Force Cambridge Research Center, under Contract No. AF-19(122)-42. It is published for technical information only, and does not necessarily represent recommendations or conclusions of the sponsoring agency.

New York, 1953

Abstract

The diffraction of a plane wave by an infinite grating of identical circular cylinders is considered. It is shown that the various scattered cylindrical waves combine to form a finite number of plane waves and an infinite number of "surface waves" propagated along the grating.

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I. Introduction

We consider the diffraction of a plane wave by an infinite grating of identical circular cylinders. The total scattered field can be represented as a series which is the sum of waves scattered by the individual cylinders. We shall not be concerned with the coefficients in the representations of these waves, but rather consider only the summation of the waves over the individual cylinders of the grating. We show that the scattered cylindrical waves combine to form a finite number of plane waves propagated in the directions given by the well-known grating diffraction formula, and in addition an infinite number of " surface waves " propagated along the grating with amplitudes which decrease exponentially with the distance from the grating.

Although the final results have been derived previously by several writers[1], our procedure, which is an extension of our earlier work on the finite grating[2], yields a direct derivation of the mode representation from the cylindrical wave representation.

II. Perpendicular Incidence

We consider a grating consisting of an infinite number of infinitely long cylinders of equal circular cross-section, arranged along the y-axis at equal intervals γ (" grating constant ").

A plane wave

(1) $$\psi^{(i)} = a_o \exp(-i\omega t + ikx),$$

where

(1a) $$k = \omega/c = 2\pi/\lambda,$$

impinges perpendicularly on the grating (see Figure 1). The response of the cylinder s (s runs from $-\infty$ to $+\infty$) can be written in the following form:

(2) $$\psi_s = \sum_{n=-\infty}^{n=+\infty} i^n A_n \cdot H_n^{(1)}(kr_s) \exp(in\theta_s) \cdot \exp(-i\omega t) .$$

Here $H_n^{(1)}$ is the Hankel function of the first kind of order n; r_s is the distance of the field point P from the center of the cylinder s; θ_s is the angle between the vector $\vec{r}_s$ and the positive x-axis. The coefficients A_n are determined by the boundary condition at the surface of the cylinders; they are independent

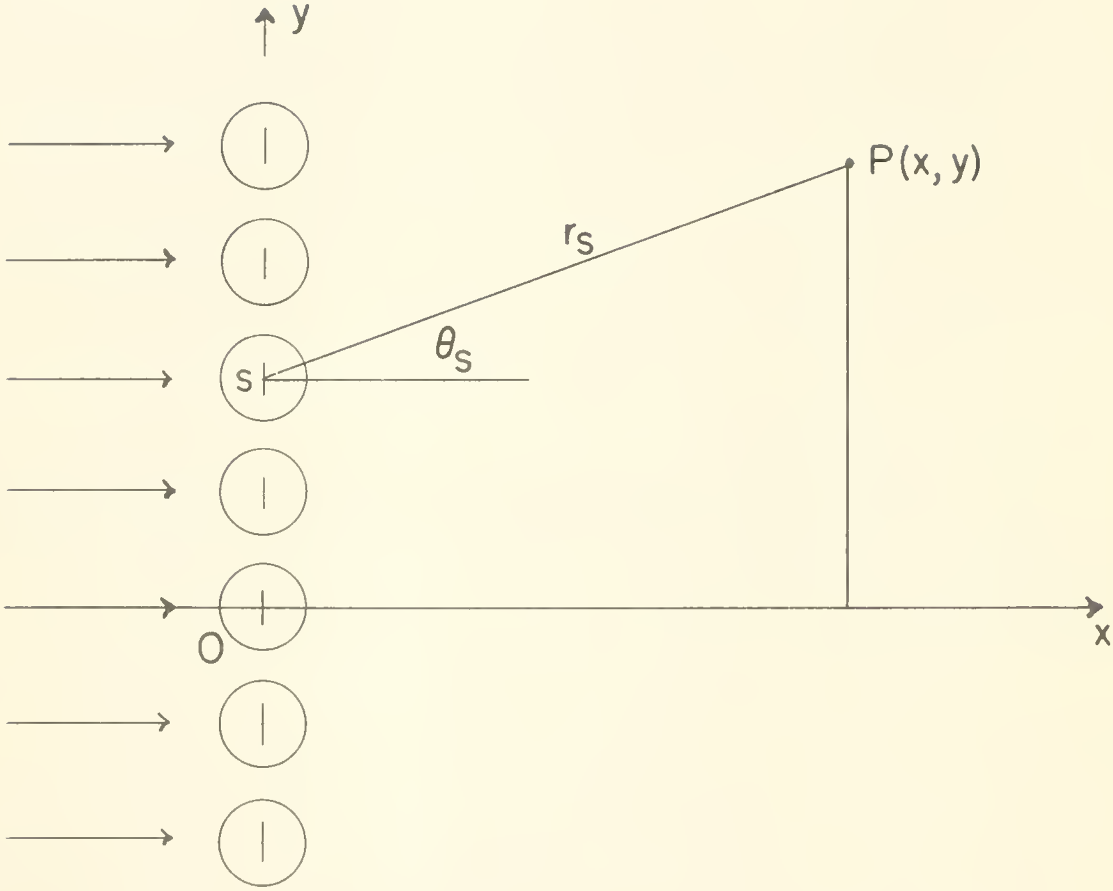

Figure 1: Wave incident on the infinite grating. Normal incidence.

of s, since the grating is infinite. In papers by V. Twersky [3] and S. Karp [4], in which the effects of multiple scattering are taken into account, these coefficients are discussed. In the present paper we need not specify the A_n since we are concerned only with carrying out the summation over s explicitly.

From Figure 1 we see that

$$(3) \qquad r_s = \sqrt{x^2 + (y - s\gamma)^2}$$

$$\tan \theta_s = \frac{y - s\gamma}{x} \quad ,$$

where x and y are the coordinates of the field-point P. We consider the field only for positive x. The total scattered wave is

$$(4) \qquad \Psi^{(scat)} = e^{-i\omega t} \cdot S,$$

where

$$(4a) \qquad S = \sum_{s=-\infty}^{s=+\infty} \sum_{n=-\infty}^{n=+\infty} i^n A_n H_n^{(1)} \left(k \sqrt{x^2 + (y - s\gamma)^2}\right) \cdot \exp(in\theta_s) .$$

Because the number of cylinders in the grating is infinite, S must be periodic in y, with the period γ. Hence

$$(5) \qquad S = \sum_{m=-\infty}^{+\infty} C_m(x) \exp\left(\frac{2\pi imy}{\gamma} \right) ,$$

where

$$(5a) \qquad C_m(x) = \frac{1}{\gamma} \cdot \int_{-\frac{\gamma}{2}}^{+\frac{\gamma}{2}} S \cdot \exp\left(- \frac{2\pi imy}{\gamma} \right) dy .$$

We now insert the expression (4a) for S into (5a); then, taking into account (3) and introducing the new variable $\eta = y - s\gamma$, we get

$$C_m(x) = \frac{1}{\gamma} \sum_{s=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} i^n A_n \cdot \int_{-\frac{\gamma}{2}-s\gamma}^{+\frac{\gamma}{2}-s\gamma} d\eta \cdot H_n^{(1)} \left(k \sqrt{x^2 + \eta^2}\right)$$

$$\cdot \exp\left(in \arctan\left(\frac{\eta}{x}\right) - \frac{2\pi im\eta}{\gamma}\right),$$

or, equivalently,

(6) $$C_m(x) = \frac{1}{\gamma}\sum_{n=-\infty}^{+\infty} i^n A_n \cdot \int_{-\infty}^{+\infty} d\eta \cdot H_n^{(1)}\left(k\sqrt{x^2+\eta^2}\right)$$

(6) $$\cdot \exp\left(i\,n \arctan\left(\frac{\eta}{x}\right) - \frac{2\pi i m \eta}{\gamma}\right).$$

We introduce $\frac{\eta}{x} = \xi$ and use

$$\arctan \xi = \frac{1}{2i}\log\left(\frac{1+i\xi}{1-i\xi}\right) \quad ;$$

therefore

$$\exp\left(i n \arctan \xi\right) = \left(\frac{1+i\xi}{\sqrt{1+\xi^2}}\right)^n ,$$

and we obtain finally:

(7) $$C_m(x) = \frac{x}{\gamma}\sum_{n=-\infty}^{+\infty} i^n A_n P_n$$

where

$$P_n = \int_{-\infty}^{+\infty} d\xi \cdot H_n^{(1)}\left(q\sqrt{1+\xi^2}\right)\left(\frac{1+i\xi}{\sqrt{1+\xi^2}}\right)^n \cdot e^{-ip\xi} .$$

Here

(7a) $$q = kx \quad ; \quad p = \frac{2\pi m x}{\gamma} .$$

For $n = 0$, one has

$$P_0 = 2\cdot\int_0^{\infty} d\xi \cdot H_0^{(1)}\left(q\sqrt{1+\xi^2}\right)\cos(p\xi)$$

(8) $$= \frac{2}{\sqrt{q^2-p^2}}\exp\left(i\sqrt{q^2-p^2}\right), \qquad \text{for } |q| > |p|$$

$$= -\frac{2i}{\sqrt{p^2-q^2}}\exp\left(-\sqrt{p^2-q^2}\right), \qquad \text{for } |p| > |q|$$

(see [5], p. 47).

In order to compute P_n for $n > 0$, we write

$$P_n = \sum_{r=0}^{r=n} (-1)^r \binom{n}{r} \frac{d^r Q_n}{dp^r} \tag{9}$$

where

$$Q_n = \int_{-\infty}^{+\infty} d\xi \cdot \frac{H_n^{(1)}\left(q\sqrt{1+\xi^2}\right)}{(1+\xi^2)^{n/2}} \cdot e^{-ip\xi} , \tag{10}$$

that is,

$$Q_n = 2 \cdot \int_0^{\infty} d\xi \cdot \frac{H_n^{(1)}\left(q\sqrt{1+\xi^2}\right)}{(1+\xi^2)^{n/2}} \cdot \cos(p\xi) . \tag{10a}$$

This integral can easily be found from a general formula given by Magnus and Oberhettinger ([5], p. 53). We have

$$\begin{aligned} Q_n &= \frac{\sqrt{2\pi}}{q^n} \cdot \left(\sqrt{q^2-p^2}\right)^{n-\frac{1}{2}} \cdot H^{(1)}_{n-1/2}\left(\sqrt{q^2-p^2}\right), \quad \text{if } |q| > |p| \\ Q_n &= \frac{\sqrt{2\pi}}{q^n} \cdot i^{n-\frac{1}{2}} \cdot \left(\sqrt{p^2-q^2}\right)^{n-\frac{1}{2}} \cdot H^{(1)}_{n-1/2}\left(i\sqrt{p^2-q^2}\right), \text{ if } |p| > |q| . \end{aligned} \tag{11}$$

From (9) and (11) it follows that

$$P_o = Q_o , \tag{12}$$

which is in agreement with (8).

For $n = 1,2,3\ldots.$ computations based on (9) and (11) yield:

$$\begin{aligned} P_1 &= \frac{2}{q} e^{i\sigma} \left(\frac{p}{\sigma} - i\right) \\ P_2 &= \frac{2}{q^2} e^{i\sigma} \cdot \sigma \left(\frac{p}{\sigma} - i\right)^2 \\ P_3 &= \frac{2}{q^3} e^{i\sigma} \cdot \sigma^2 \left(\frac{p}{\sigma} - i\right)^3 \\ &\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned} \tag{13}$$

for $|q| > |p|$,

where

$$\sigma = \sqrt{q^2 - p^2} \quad . \tag{14}$$

For $|q| < |p|$ we obtain

$$P_1 = -\frac{2i}{q} \cdot e^{-\sigma'} \left(\frac{p}{\sigma'} + 1 \right)$$

$$P_2 = -\frac{2i}{q^2} e^{-\sigma'} \cdot \sigma' \left(\frac{p}{\sigma'} + 1 \right)^2 \tag{13a}$$

$$P_3 = -\frac{2i}{q^3} e^{-\sigma'} \cdot \sigma'^2 \left(\frac{p}{\sigma'} + 1 \right)^3$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots ,$$

where

$$\sigma' = \sqrt{p^2 - q^2} \quad . \tag{14a}$$

The results (13) and (13a) may be generalized immediately as:

$$P_n = \frac{2}{q^n} e^{i\sigma} \cdot \sigma^{n-1} \cdot \left(\frac{p}{\sigma} - i \right)^n , \qquad \text{for } |q| > |p| \; ; \tag{15}$$

$$P_n = -\frac{2i}{q^n} e^{-\sigma'} \cdot \sigma'^{n-1} \cdot \left(\frac{p}{\sigma'} + 1 \right)^n , \qquad \text{for } |q| < |p| \; . \tag{15a}$$

As a matter of fact, the definition (7) of P_n leads to the recurrence formula

$$\frac{\partial P_n}{\partial q} + \frac{n}{q} P_n = P_{n-1} - \frac{\partial P_{n-1}}{\partial p} \tag{16}$$

if one uses the relation

$$H_n'(z) = -\frac{n}{z} H_n(z) + H_{n-1}(z) \; . \tag{17}$$

For the case $|q| > |p|$, let us now assume that the following relation holds:

$$(18) \qquad q^{n-1} \cdot P_{n-1} = 2 \cdot \frac{e^{i\sigma}}{\sigma} \cdot (p - i\sigma)^{n-1} ,$$

with

$$\sigma = \sqrt{q^2 - p^2}$$

(see eq. (15)). Then, from (16) and (18) we have

$$\frac{\partial}{\partial q}\left[q^n P_n\right] = 2q \, \frac{e^{i\sigma}}{\sigma^2} (p - i\sigma)^{n-1} \cdot \left\{\sigma - \frac{p}{\sigma} + i(p + 1 - n)\right\}$$

(19) or

$$\frac{\partial}{\partial \sigma}\left[q^n P_n\right] = 2 \, \frac{e^{i\sigma}}{\sigma} (p - i\sigma)^{n-1} \cdot \left\{\sigma - \frac{p}{\sigma} + i(p + 1 - n)\right\} .$$

The relation (19) is indeed valid if we put

$$(20) \qquad q^n P_n = 2 \, \frac{e^{i\sigma}}{\sigma} \, (p - i\sigma)^n .$$

Since P_0 is known from (8), and since we have just shown that (15) is true for n+1 if it is true for n, it follows by mathematical induction that (15) is true for all values of n. Equation (15a) can be proven in the same way.

In order to evaluate S as given by (5) we must consider the two cases $|q| \gtrless |p|$ separately.

1. Case $|q| > |p|$. Introducing the diffraction angle α by the relation:

$$(21) \qquad \sin \alpha_m = \frac{2\pi m}{k\gamma} = m \frac{\lambda}{\gamma} = \frac{p}{q}$$

(see (7a)), we can write P_n in the form

$$(22) \qquad P_n = \frac{2}{kx} \exp(ikx \cos \alpha_m) \cdot (\cos \alpha_m)^{n-1} \cdot (\tan \alpha_m - i)^n ,$$

which is valid for positive and negative m, including $m = 0$, subject to the condition $|m| < \frac{\gamma}{\lambda}$. Inserting (22) into (7) and the resulting expression for $C_m(x)$ into (5), we can write S as follows:

$$S = \frac{2}{k\gamma} \sum_{|m| < \frac{\gamma}{\lambda}} \frac{1}{\cos \alpha_m} \cdot \exp\left(ik\left\{x \cos \alpha_m + y \sin \alpha_m\right\}\right) \cdot \sum_{n=-\infty}^{+\infty} A_n \cdot \exp(in\alpha_m) \ . \tag{23}$$

Reintroducing the time factor $e^{-i\omega t}$ [see (4)], we see that the scattered wave appears as a superposition of a finite number of plane waves propagated along the directions given by the usual grating formula (21). The amplitudes of these waves are:

$$B_m = \frac{2}{k\gamma \cos \alpha_m} \sum_{n=-\infty}^{+\infty} A_n \cdot \exp(in\alpha_m) \, . \tag{24}$$

Besides these plane waves there still exists an infinite number of scattered waves of a quite different type. These correspond to the case $|p| > |q|$.

2. Case $|p| > |q|$. We put

$$\frac{p}{q} = m\frac{\lambda}{\gamma} = \pm \cosh \beta_m \, , \tag{25}$$

where $|m| \geqq \frac{\gamma}{\lambda}$ (the (+) sign is taken for m positive and the (-) sign for m negative). Then from (5), (7) and (15a) we obtain

$$S = -\frac{2i}{k\gamma}\left\{\sum_{m > \gamma/\lambda}^{m=+\infty} \frac{\exp(-kx \sinh \beta_m)}{\sinh \beta_m} \cdot \exp(iky \cosh \beta_m) \sum_{n=-\infty}^{n=+\infty} i^n A_n \exp(n\beta_m) \right. \tag{26}$$

$$\left. + \sum_{m=-\infty}^{|m|>\gamma/\lambda} \frac{\exp(-kx \sinh \beta_m}{\sinh \beta_m} \cdot \exp(-iky \cosh \beta_m) \sum_{n=-\infty}^{n=+\infty} (-i)^n A_n \cdot \exp(-n\beta_m)\right\} .$$

where in the first sum over m, m is positive, and in the second, m is negative.

Reintroducing the time factor $e^{-i\omega t}$, we see that the scattered waves, infinite in number, are "surface waves" propagated along the grating in both y-directions. The spatially damped "amplitudes" of these surface waves are

$$B_m' = -\frac{2i}{k\gamma} \cdot \frac{\exp(-kx \sinh \beta_m)}{\sinh \beta_m} \cdot \sum_{n=-\infty}^{+\infty} i^n A_n \cdot \exp(n\beta_m), \qquad \text{for positive } m$$

(27)

$$B_m'' = -\frac{2i}{k\gamma} \cdot \frac{\exp(-kx \sinh \beta_m}{\sinh \beta_m} \cdot \sum_{n=-\infty}^{+\infty} (-i)^n A_n \cdot \exp(-n\beta_m), \qquad \text{for negative } m$$

in the directions (+y) and (-y) respectively. The amplitudes B_m'' and B_m' are equal if

(28) $$A_{-n} = A_n .$$

This is certainly the case if the cross-sections of the cylinders are circular and if the cylinders are perfect conductors. Then we have

(29) $$A_n = -\frac{J_n(ka)}{H_n^{(1)}(ka)}$$

(a is the radius of the cylinder).

We should note that for $|m| = \frac{\gamma}{\lambda}$ both the factor $\cos \alpha_m$ in the denominator of B_m (see (24)) and the factor $\sinh \beta_m$ in the denominator of B_m' and B_m'' (see (27)) become zero, because in this case $\alpha_m = \frac{\pi}{2}$, $\beta_m = 0$. This would make the amplitudes B_m, B_m', B_m'' infinite. It has been shown by Twersky and Karp ([3] and [4]) that this "resonance difficulty" is a result of neglecting multiple scattering.

III. Oblique Incidence

It is easy to extend the above discussion to the case of oblique incidence of the primary wave. Let us assume (see Figure 2) that the primary wave is incident on the grating at an angle $(-\beta)$ with the x-axis. Then

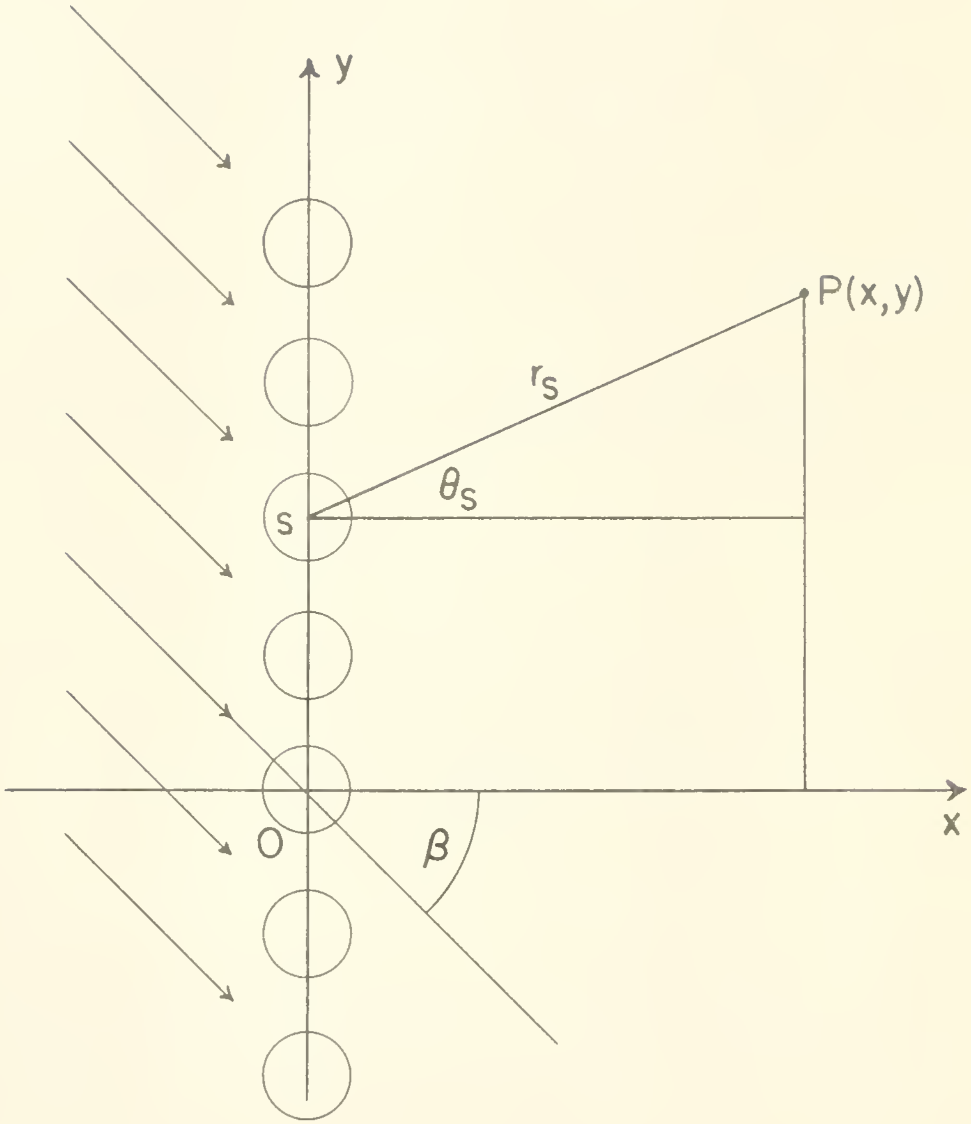

Figure 2: Wave incident on the infinite grating. Oblique incidence.

$$\psi^{(i)} = a_0 \, e^{-i\omega t + ik(x\cos\beta - y\sin\beta)} \tag{30}$$

and the total scattered wave becomes

$$\Psi^{(scat)} = e^{-i\omega t} \cdot S \tag{31}$$

where

$$S = \sum_{s=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} i^n A_n e^{in\beta} \cdot H_n^{(1)}\left(k\sqrt{x^2+(y-s\gamma)^2}\right) \exp(in\theta_s - iks\gamma\sin\beta) \,.$$

Again, θ_s is the angle which r_s makes with the x-axis, so that

$$\tan\theta_s = \frac{y - s\gamma}{x} \,. \tag{32}$$

In analogy with (5), we now have

$$S = e^{-iky\sin\beta} \cdot \sum_{m=-\infty}^{m=+\infty} C_m(x) \exp\left(\frac{2\pi imy}{\gamma}\right) \tag{33}$$

with

$$C_m(x) = \frac{1}{\gamma} \cdot \int_{-\frac{\gamma}{2}}^{+\frac{\gamma}{2}} S \cdot \exp\left(iky\sin\beta - \frac{2\pi imy}{\gamma}\right) \cdot dy \,. \tag{34}$$

We insert S from (31) into (34); then, taking into account (32), and introducing the new variable

$$\xi = \frac{y - s\gamma}{x} \tag{35}$$

we obtain finally

$$C_m(x) = \frac{x}{\gamma} \sum_{n=-\infty}^{+\infty} i^n A_n \, e^{in\beta} \cdot \overline{P}_n \,, \tag{36}$$

where

$$\overline{P}_n = \int_{-\infty}^{+\infty} d\xi \cdot H_n^{(1)}\left(q\sqrt{1+\xi^2}\right)\left(\frac{1+i\xi}{\sqrt{1+\xi^2}}\right)^n \cdot e^{-ip'\xi} \,. \tag{37}$$

Here,

$$(38)\quad \begin{cases} q = kx \;;\; p' = x\left\{\dfrac{2\pi m}{\gamma} - k \sin\beta\right\} , \\ \text{that is,} \\ p' = p - q\sin\beta = q\left(\dfrac{p}{q} - \sin\beta\right) , \end{cases}$$

where p is defined by (7a).

The only difference between (7) and (37) is that p has been replaced by p'. Hence from (15) we may conclude immediately that

$$(39)\quad \begin{cases} \bar{P}_n = \dfrac{2}{q^n}\, e^{i\bar{\sigma}} \cdot \bar{\sigma}^{n-1} \cdot \left(\dfrac{p'}{\bar{\sigma}} - i\right)^n & \text{for } |q| > |p'| , \\ \bar{P}_n = -\dfrac{2i}{q^n}\, e^{-\bar{\sigma}'} \cdot \bar{\sigma}'^{\,n-1} \cdot \left(\dfrac{p'}{\bar{\sigma}'} + 1\right)^n & \text{for } |q| < |p'| , \end{cases}$$

where

$$(39a)\qquad \bar{\sigma} = \sqrt{q^2 - p'^2} \;; \qquad \bar{\sigma}' = \sqrt{p'^2 - q^2}$$

Again, in order to evaluate S we consider the two cases $|q| \gtrless |p'|$ separately.

1. Case $|q| > |p'|$. We put, in analogy with (21):

$$(40)\qquad \frac{p'}{q} = \frac{2\pi m}{k\gamma} - \sin\beta = \sin\bar{\alpha}_m .$$

so that

$$(40a)\qquad \sin\bar{\alpha}_m + \sin\beta = m\frac{\lambda}{\gamma} .$$

Then $|q| > |p'|$ implies that

$$(41)\qquad 1 > \left|\frac{2\pi m}{k\gamma} - \sin\beta\right| ,$$

that is,

$$|\sin\bar{\alpha}_m| < 1 .$$

From (36), (39) and (33) we find

$$(42) \qquad S = \frac{2}{k\gamma}\sum_{m} \frac{1}{\cos\bar{\alpha}_m} \cdot \exp\left(ik\left\{x\cos\bar{\alpha}_m + y\sin\bar{\alpha}_m\right\}\right)\sum_{n=-\infty}^{+\infty} A_n \cdot \exp\left[in(\bar{\alpha}_m+\beta)\right] .$$

The values of m range from $m \geqq -\frac{\gamma}{\lambda}(1-\sin\beta)$ to $m \leqq \frac{\gamma}{\lambda}(1+\sin\beta)$.

Reintroducing the time-factor $e^{-i\omega t}$ we see that the scattered wave appears as a superposition of a finite number of plane waves propagated along the directions given by the usual grating fromula (40a). The angle $\bar{\alpha}_m$ is the "diffraction angle", that is, the angle between the x-axis (the normal to the grating) and the propagation vector of the diffracted waves. The amplitudes of these waves are:

$$(43) \qquad \bar{B}_m = \frac{2}{k\gamma\cos\bar{\alpha}_m} \cdot \sum_{n=-\infty}^{+\infty} A_n \cdot \exp\left[in(\bar{\alpha}_m + \beta)\right] .$$

2. Case $|q| < |p'|$. Put

$$(44) \qquad \frac{p'}{q} = m\frac{\lambda}{\gamma} - \sin\beta = \pm\cosh\bar{\beta}_m ,$$

where the (+) sign is taken for $m > \frac{\gamma}{\lambda}(1+\sin\beta)$, and the (-) sign for $m < -\frac{\gamma}{\lambda}(1-\sin\beta)$. Then from (33), (36) and (39) we obtain

$$S = -\frac{2i}{k\gamma}\left\{\sum_{m>\frac{\gamma}{\lambda}(1+\sin\beta)} \cdot \frac{\exp(-kx\sinh\bar{\beta}_m)}{\sinh\bar{\beta}_m} \cdot \exp(iky\cosh\bar{\beta}_m)\right.$$

$$(45) \qquad \cdot\sum_{n=-\infty}^{+\infty} i^n A_n \exp\left[(i\beta+\bar{\beta}_m)n\right] + \sum_{m<\gamma/\lambda(1-\sin\beta)} \frac{\exp(-kx\sinh\bar{\beta}_m)}{\sinh\bar{\beta}_m}$$

$$\left. \cdot \exp(iky\cosh\bar{\beta}_m)\sum_{n=-\infty}^{+\infty}(-i)^n \cdot A_n \exp\left[(i\beta-\bar{\beta}_m)n\right]\right\} .$$

This is the generalization of (26) for oblique incidence of the primary wave.

Reintroducing the time factor $e^{-i\omega t}$, we see that the scattered waves, infinite in number, are "surface waves" propagated along the grating in both y-directions. The mode characterized by m propagates with the velocity

$$V_m = \frac{c}{\left| m \frac{\lambda}{\gamma} - \sin \beta \right|} , \tag{46}$$

where the denominator is greater than 1. Its wave length is

$$\Lambda_m = \frac{\lambda}{\left| m \frac{\lambda}{\gamma} - \sin \beta \right|} \tag{47}$$

and its amplitude

$$\overline{B_m'} = -\frac{2i}{k\gamma} \cdot \frac{\exp(-kx \sinh \bar{\beta}_m)}{\sinh \bar{\beta}_m} \cdot \sum_{n=-\infty}^{+\infty} i^n A_n \exp\left[(i\beta + \bar{\beta}_m)n\right], \quad \text{for positive m,}$$

(48)

$$\overline{B_m''} = -\frac{2i}{k\gamma} \cdot \frac{\exp(-kx \sinh \bar{\beta}_m)}{\sinh \bar{\beta}_m} \cdot \sum_{n=-\infty}^{+\infty} (-i)^n \cdot A_n \exp\left[(i\beta - \bar{\beta}_m)n\right], \quad \text{for negative m.}$$

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Date Due

OCT 7 1955

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JUN 1 '60

JAN 11 '68

ILL - 1.7.93

PRINTED IN U. S. A.

NYU, D.Em.R., Res. rep. EM-61
AUTHOR
Reiche, R.
TITLE
On diffraction by an infinite grating.

DATE LOANED	BORROWER'S NAME	ROOM NUMBER
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Manufactured in the United States for New York University Press
by the University's Office of Publications and Printing